



I'm **Brad Rodgers**, an Associate Professor here in the Department of Math and Stats at Queen's University.

My research is largely in **analytic number theory** and **random matrix and point process theory**.

Problems I am interested in often involve applying ideas from functional and harmonic analysis and probability to number theory. Two examples of topics I have been interested in recently are:

**The distribution of arithmetic functions in short intervals** In a first course in number theory one often studies the average value of arithmetic functions or sequences on *deterministic long intervals*; for instance the prime number theorem is the statement that the likelihood a random number  $n$  in between 1 and  $X$  is prime is roughly  $1/\log X$ .

What can be said about the number of primes in *random short interval*  $[n, n + X^{1/4}]$  where  $n$  is again chosen uniformly from 1 to  $X$ ? Such a question is more subtle than the prime number theorem – this question in particular is closely connected to the fine-scale distribution of zeros of the Riemann zeta-function. Similar questions for sums of divisor functions over random short intervals are related to moments of the Riemann zeta-function. In recent work collaborators and I have answered an old question of this sort about the number of squarefree integers in random short intervals. The problem ends up being related to questions of the following sort: count the number of reduced rational solutions to

$$a_1/q_1 + a_2/q_2 + a_3/q_3 + a_4/q_4 = 0$$

where  $|q_i| \leq Q$  and  $|a_i/q_i| \leq Q^{-1/10}$ . Are most solutions given by the obvious 'paired' solutions, in which e.g.  $a_1/q_1 = -a_2/q_2$  and  $a_3/q_3 = -a_4/q_4$ ?

**The distribution of random or special trigonometric polynomials** An old theorem Salem and Zygmund says that for a "typical" choice of independent random coefficients  $\epsilon_1 = \pm 1, \epsilon_2 = \pm 1, \dots$

$$\sup_{\theta} \left| \sum_{n=1}^N \epsilon_n e^{in\theta} \right| \asymp \sqrt{N \log N},$$

as  $N$  grows. What happens when the coefficients  $\epsilon_n$  are chosen in a more structured way? For instance I would like to know if the same is true when  $\epsilon_n$  is a random multiplicative function. One can also ask extremal questions. It is known that there exists choices of  $\epsilon_n$  such that the above quantities are  $O(\sqrt{N})$ . (Parseval tells us this is the smallest one could hope for.) Can anything sensible be said when the coefficients  $\epsilon_n$  are restricted to be a multiplicative function?