DEPARTMENT OF MATHEMATICS AND STATISTICS
Summer 2025 Research Projects (in no particular order)

Queen's University

Last updated: 15 January 2025

Project Title: Markov genealogy processes

Supervisor: Felicia Magpantay and Troy Day

Project Description: During an epidemic the emergence of "variants of concern" is shaped by evolutionary pressures such as vaccination and competition. The pandemic has led to an explosion in genomic surveillance, highlighting the need for efficient and mathematically rigorous analysis of epidemiological time series coupled with genetic data. In this project we will look into the recently developed theory of Markov genealogy processes. We will also examine the implementation of this theory for statistical inference in the R package phylopomp and apply it to datasets.
Main reference: https://doi.org/10.1016/j.tpb.2021.11.003

Student's Role: The student will participate in our study of Markov genealogy processes and conduct a review of different approaches in phylodynamic analysis. The student will also learn about phylopomp and high-performance computing, and apply these to a disease modeling project using real data.

Prerequisites (req'd background/level of study): Solid background in probability, basic mathematical or statistical programming.

Project Title: Mathematics of Reinforcement Learning under Partial Information

Supervisor: Serdar Yuksel

Project Description: For many stochastic decision and control problems, an exact model of a system being studied is not available; and even when an approximate or exact model is available, the information at the decision maker with regard to the state being controlled is often limited and partial. Data-driven learning is an appropriate framework for such settings. The general goal of this project is to study the optimal design of decision/control policies in such stochastic systems with incomplete models and partial information via reinforcement or empirical learning methods. Depending on student interest, we will focus on one of the following, closely related, three main themes:

- (i) Optimal stochastic control with partial information: Non-linear filtering, approximations, and relations with reinforcement learning
- (ii) Multi-agent systems, stochastic teams, and games: Arriving at optimality/equilibrium under decentralized information
- (iii) Sample complexity in such learning algorithms

Student's Role: The expectation is that the student would study a wide variety of resources that will be provided, take part in regular research discussions through frequent meetings, and write a mathematically rigorous technical document at the end of the summer.

Prerequisites (req'd background/level of study): A strong foundation and interest in probability theory and analysis.

Project Title: Investigating Secant Varieties

Supervisors: Mike Roth and Gregory G. Smith.

Project Description: A projective variety X is the common zero-locus of a family of homogeneous polynomial equations. The associated secant variety is the closure of the union of all secant lines to X; this new space is again described by polynomial equations. When X has dimension n, one expects that its secant variety will have dimension 2n+1, because the two points of contact on the secant line each move in an n-dimensional family (namely X) and the line adds one extra dimension. Embedded varieties where the dimension of a secant variety is strictly less than the expect dimension are called defective. Remarkably, the only algebraic variety where the defectivity of secants is understood for all embeddings is when X is itself a projective space. The goal of this project is to investigate the problem of defective secants when X is a toric variety. Toric varieties are particularly nice class of varieties with a strong connection to combinatorics and convex polytopes.

Student's Role: To learn some of the dictionary between the algebraic, geometric, and combinatorial ideas, as well as some basic background in geometry relevant to the problem. To use a computer to gather data on toric varieties (especially toric surfaces) with defective secants and to formulate conjectures.

Prerequisites (req'd background/level of study): The student should at least have taken Math 110, Math 120, and Math 210. Upper-year courses in algebra or geometry are a plus. The student should be willing to use a computer to do experiments, and be willing to learn how to use the software appropriate for these computations.

Project Title: Pinning Stabilization of Complex Networks: Using Event-Triggered Impulses

Supervisor: Kexue Zhang

Project Description: Complex networks (CNs) consist of vast nodes typically modelled as dynamical systems, with specific topological structures governing their connections. Impulsive control, a distinctive feedback control, employs abrupt state changes or jumps at discrete time intervals to achieve stabilization. This impulsive control paradigm has demonstrated robustness and efficiency in facilitating network stabilization. In this project, we will investigate stabilization problems for CNs using pinning impulsive control approaches, where only a subset of the network nodes is controlled. We will explore novel event-triggering algorithms to identify the timing for impulses, and then construct sufficient conditions related to the network topology, impulsive control gains, and parameters in the event-triggering conditions that ensure network stabilization.

Student's Role: Study various stability notions and stability results for continuous-time systems and systems with impulses; explore the application of stability results for systems with impulses to event-triggered impulsive control of complex networks. Conduct numerical simulations to verify the impulsive stabilization process for complex networks.

Prerequisites (req'd background/level of study): A background in differential equations. Knowledge of Lyapunov stability and experience in MATLAB will be a plus.

Project title: Analyzing Computer Experiments with Time Series and Tensor Outputs

Supervisor: C. Devon Lin

Project description: Computer experiments are now ubiquitous in scientific research and engineering. The computer models used in computer experiments are often very complex and computationally expensive, thus requiring emulators in the analysis. Gaussian process (GP) models, a.k.a. kriging, have been used as a core tool for modeling computer experiments. The conventional GP models often only consider the univariate response, while many practical applications have time series (vector-valued) responses or tensor responses (matrix-valued or high dimensional) Constructing desirable emulators for such computer experiments remains a challenging problem. A few methods are proposed, which include dynamic Gaussian processes and a newly proposed approach. The R package DynamicGP has been built to build statistical models for computer experiments with time series responses. However, it does not consider the inference problems such as optimization and contour estimation. In this project, we will implement the new methods in the standard statistical software, R, update and build a R package for the easy and convenient use of researchers and practitioners.

Student's role: The student is expected to do a literature review, understanding existing R codes, implement some methods in R, building a R package and writing a documentation. The student will gain a deeper understanding of statistical concepts, modeling, and become expertise in R programming. The student will be the author of the R package and an article sent to Journal of Statistical Software.

Prerequisites (req'd background/level of study): Be familiar with R, have a strong background in computing and statistics.

Project Title: Quotients of root systems of infinite Coxeter groups and inversion sets

Supervisors: Ivan Dimitrov, Charles Paquette and David Wehlau

Project Description: The group of permutations on n letters is ubiquitous in mathematics and is the simplest example of a Coxeter group. Coxeter groups are groups generated by reflections subject to braid-like relations. Many problems in modern commutative algebra and representation theory have their solutions described in terms of sets of elements of various Coxeter groups.

In previous very successful USRA projects we studied a new way to provide a recursive description of elements of finite Coxeter groups associated with their respective inversion sets. These project yielded a comprehensive solution to a very important open problem in algebra and geometry. In the current project we will investigate a broad range of important properties of elements of infinite Coxeter groups from the view point of decomposition. We expect that this project, like the previous projects, will lead to a publication.

Student's role: Generate experimental data, form conjectures, verify such conjectures with more experiments, prove them or find counter-examples.

Prerequisites (req'd background/level of study: MATH 110, MATH 210; MATH 310 and/or MATH 314 as well as familiarity with a computer algebra system would be strong assets.

Be sure to check back as additional projects may be added.